Time Delay Interferometry with Phase Locked Lasers

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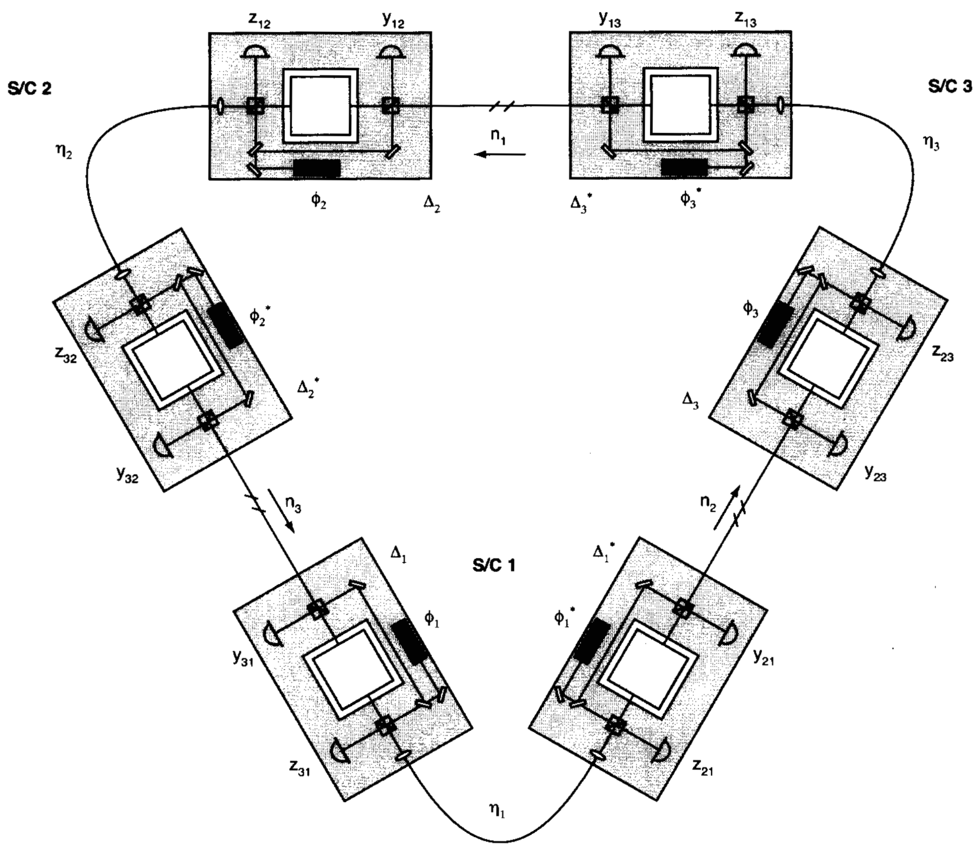

Figure 1: Optical layout of LISA interferometer showing six optical benches joined by three optical fibers and three free-space laser links.

Figure shows the six optical benches each housing two photodetectors. The phase of the interference at these photodetectors is given in equations (1)-(12) where we have taken the phase to be equal to the local laser phase minus the remote laser phase.

$$
\begin{aligned}
y_{21} &= \phi_1^* - \phi_{3,2} + \hat{n}_2 \cdot (\Delta_1^* + \Delta_{3,2}) + gw_2 & (1)\\
z_{21} &= \phi_1^* - \phi_1 + 2\hat{n}_3 \cdot \Delta_1 - \eta_1 & (2)\\
y_{31} &= \phi_1 - \phi_{2,3}^* - \hat{n}_3 \cdot (\Delta_1 + \Delta_{2,3}^*) + gw_3 & (3)\\
z_{31} &= \phi_1 - \phi_1^* - 2\hat{n}_2 \cdot \Delta_1^* - \eta_1 & (4)
\end{aligned}
$$

$$
\begin{aligned}
y_{32} &= \phi_2^* - \phi_{1,3} + \hat{n}_3 \cdot (\Delta_2^* + \Delta_{1,3}) + gw_3 & (5)\\
z_{32} &= \phi_2^* - \phi_2 + 2\hat{n}_1 \cdot \Delta_2 - \eta_2 & (6)\\
y_{12} &= \phi_2 - \phi_{3,1}^* - \hat{n}_1 \cdot (\Delta_2 + \Delta_{3,1}^*) + gw_1 & (7)\\
z_{12} &= \phi_2 - \phi_2^* - 2\hat{n}_3 \cdot \Delta_2^* - \eta_2 & (8)
\end{aligned}
$$

$$
\begin{aligned}
y_{13} &= \phi_3^* - \phi_{2,1} + \hat{n}_1 \cdot (\Delta_3^* + \Delta_{2,1}) + gw_1 & (9)\\
z_{13} &= \phi_3^* - \phi_3 + 2\hat{n}_2 \cdot \Delta_3 - \eta_3 & (10)\\
y_{23} &= \phi_3 - \phi_{1,2}^* - \hat{n}_2 \cdot (\Delta_3 + \Delta_{1,2}^*) + gw_2 & (11)\\
z_{23} &= \phi_3 - \phi_3^* - 2\hat{n}_1 \cdot \Delta_3^* - \eta_3 & (12)
\end{aligned}
$$

These expressions include laser phase information, ϕ_i, the individual displacements of the optical benches, Δ_i, phase shifts introduced by the optical fibers, η_i, and the gravitational wave phase shift gw_i. Shot noise is currently not included although it should be transfered in the same manner as the GW signal. In order to cancel the fiber noise, η_i it is necessary to only take combinations of $z_{xi} - z_{yi}$ at the same time. Thus we can combine the z_{ij}'s on the same spacecraft to form new variables $z_j = z_{xj} - z_{yj}$ and reduce the total number of data streams from twelve to nine.

$$
\begin{aligned}
z_1 \equiv \; & z_{21} - z_{31} = & 2(\phi_1^* - \phi_1 + \hat{n}_3 \cdot \Delta_1 + \hat{n}_2 \cdot \Delta_1^*) \quad (13)\\
z_2 \equiv \; & z_{32} - z_{12} = & 2(\phi_2^* - \phi_2 + \hat{n}_1 \cdot \Delta_2 + \hat{n}_3 \cdot \Delta_2^*) \quad (14)\\
z_3 \equiv \; & z_{21} - z_{23} = & 2(\phi_3^* - \phi_3 + \hat{n}_2 \cdot \Delta_3 + \hat{n}_1 \cdot \Delta_3^*) \quad (15)
\end{aligned}
$$

1 Phase locked transponders

If the lasers are phase locked in a coherent chain then the phase locking will drive various signals to zero. For the purposes of this discussion we define the phase locking hierarchy to be,

$$
\begin{aligned}
2 \rightarrow 2^* \rightarrow 1 \rightarrow 1^* \quad (16)\\
3^* \rightarrow 3 \rightarrow 1^* \quad (17)
\end{aligned}
$$

where the arrow points from slave to master laser. In this configuration laser 1^* is the ultimate master. These phase locking loops are enforced by feeding back to the appropriate laser phase to null the values of the following five observables.

$$z_1 \rightarrow 0 \quad \text{via feedback to } \phi_1 \tag{18}$$
$$y_{32} \rightarrow 0 \quad \text{via feedback to } \phi_2^* \tag{19}$$
$$z_2 \rightarrow 0 \quad \text{via feedback to } \phi_2 \tag{20}$$
$$y_{23} \rightarrow 0 \quad \text{via feedback to } \phi_3 \tag{21}$$
$$z_3 \rightarrow 0 \quad \text{via feedback to } \phi_3^* \tag{22}$$
$$\tag{23}$$

The feedback imposes the following relationships between the phases of the slave lasers and the master laser, 1^*.

$$\begin{aligned} z_1 &= 2(\phi_1^* - \phi_1 + \hat{n}_3 \cdot \Delta_1 + \hat{n}_2 \cdot \Delta_1^*) = 0 \\ \Rightarrow \phi_1 &= \phi_1^* + \hat{n}_3 \cdot \Delta_1 + \hat{n}_2 \cdot \Delta_1^* \end{aligned} \tag{24}$$

$$\begin{aligned} y_{32} &= \phi_2^* - \phi_{1,3} + \hat{n}_3 \cdot (\Delta_2^* + \Delta_{1,3}) + gw_3 = 0 \\ \Rightarrow \phi_2^* &= \phi_{1,3} - \hat{n}_3 \cdot (\Delta_2^* + \Delta_{1,3}) - gw_3 \\ \phi_2^* &= \phi_{1,3}^* + \hat{n}_2 \cdot \Delta_{1,3}^* - \hat{n}_3 \cdot \Delta_2^* - gw_3 \end{aligned} \tag{25}$$

$$\begin{aligned} z_2 &= 2(\phi_2^* - \phi_2 + \hat{n}_1 \cdot \Delta_2 + \hat{n}_3 \cdot \Delta_2^*) = 0 \\ \Rightarrow \phi_2 &= \phi_2^* + \hat{n}_1 \cdot \Delta_2 + \hat{n}_3 \cdot \Delta_2^* \\ \phi_2 &= \phi_{1,3}^* + \hat{n}_2 \cdot \Delta_{1,3}^* + \hat{n}_1 \cdot \Delta_2 - gw_3 \end{aligned} \tag{26}$$

$$\begin{aligned} y_{23} &= \phi_3 - \phi_{1,2}^* - \hat{n}_2 \cdot (\Delta_3 + \Delta_{1,2}^*) + gw_2 = 0 \\ \Rightarrow \phi_3 &= \phi_{1,2}^* + \hat{n}_2 \cdot (\Delta_3 + \Delta_{1,2}^*) - gw_2 \end{aligned} \tag{27}$$

$$\begin{aligned} z_3 &= 2(\phi_3^* - \phi_3 + \hat{n}_2 \cdot \Delta_3 + \hat{n}_1 \cdot \Delta_3^*) = 0 \\ \Rightarrow \phi_3^* &= \phi_3 - \hat{n}_2 \cdot \Delta_3 - \hat{n}_1 \cdot \Delta_3^* \\ \phi_3^* &= \phi_{1,2}^* + \hat{n}_2 \cdot \Delta_{1,2}^* - \hat{n}_1 \cdot \Delta_3^* - gw_2 \end{aligned} \tag{28}$$

The remaining 4 observables y_{21}, y_{31}, y_{13}, and y_{12} are recorded and processed to extract the gravitational wave signal.

$$\begin{aligned} y_{21} &= \phi_1^* - \phi_{3,2} + \hat{n}_2 \cdot (\Delta_1^* + \Delta_{3,2}) + gw_2 \\ &= \phi_1^* - \phi_{1,22}^* - \hat{n}_2 \cdot (\Delta_{3,2} + \Delta_{1,22}^*) + gw_{2,2} + \hat{n}_2 \cdot (\Delta_1^* + \Delta_{3,2}) + gw_2 \\ &= \phi_1^* - \phi_{1,22}^* + \hat{n}_2 \cdot (\Delta_1^* - \Delta_{1,22}^*) + gw_2 + gw_{2,2} \end{aligned} \tag{29}$$

$$\begin{aligned} y_{31} &= \phi_1 - \phi_{2,3}^* - \hat{n}_3 \cdot (\Delta_1 + \Delta_{2,3}^*) + gw_3 \\ &= \phi_1^* + \hat{n}_3 \cdot \Delta_1 + \hat{n}_2 \cdot \Delta_1^* - \phi_{1,33}^* - \hat{n}_2 \cdot \Delta_{1,33}^* + \hat{n}_3 \cdot \Delta_{2,3}^* + gw_{3,3} - \hat{n}_3 \cdot (\Delta_1 + \Delta_{2,3}^*) + gw_3 \\ &= \phi_1^* - \phi_{1,33}^* + \hat{n}_2 \cdot (\Delta_1^* - \Delta_{1,33}^*) + gw_3 + gw_{3,3} \end{aligned} \tag{30}$$

$$\tag{31}$$

$$
\begin{aligned}
y_{13} &= \phi_3^* - \phi_{2,1} + \hat{n}_1 \cdot (\Delta_3^* + \Delta_{2,1}) + gw_1 \\
&= \phi_{1,2}^* + \hat{n}_2 \cdot \Delta_{1,2}^* - \hat{n}_1 \cdot \Delta_3^* - gw_2 - \phi_{1,31}^* - \hat{n}_2 \cdot \Delta_{1,31}^* - \hat{n}_1 \cdot \Delta_{2,1} + gw_{3,1} + \hat{n}_1 \cdot (\Delta_3^* + \Delta_{2,1}) + gw_1 \\
&= \phi_{1,2}^* - \phi_{1,31}^* + \hat{n}_2 \cdot (\Delta_{1,2}^* - \Delta_{1,31}^*) - gw_2 + gw_{3,1} + gw_1 \qquad (32)
\end{aligned}
$$

$$
\begin{aligned}
y_{12} &= \phi_2 - \phi_{3,1}^* - \hat{n}_1 \cdot (\Delta_2 + \Delta_{3,1}^*) + gw_1 \\
&= \phi_{1,3}^* + \hat{n}_2 \cdot \Delta_{1,3}^* + \hat{n}_1 \cdot \Delta_2 - gw_3 - \phi_{1,21}^* - \hat{n}_2 \cdot \Delta_{1,21}^* + \hat{n}_1 \cdot \Delta_{3,1}^* + gw_{2,1} - \hat{n}_1 \cdot (\Delta_2 + \Delta_{3,1}^*) + gw_1 \\
&= \phi_{1,3}^* - \phi_{1,21}^* + \hat{n}_2 \cdot (\Delta_{1,3}^* - \Delta_{1,21}^*) - gw_3 + gw_{2,1} + gw_1 \qquad (33)
\end{aligned}
$$

2 Gravitational Wave Signals

To extract the gravitational wave information from these signals it is necessary to combine these four observables in specific linear combinations. In the absence of arm length mismatches (ie $L_1 = L_2 = L_3$) the appropriate combinations for the three Michelson-type signals, X_1, X_2 and X_3 are given below (ignoring z_i terms).

$$
\begin{aligned}
X_1 &= y_{21} + y_{23,2} - (y_{31} + y_{32,3}) \\
&= y_{21} - y_{31} \qquad (34) \\
X_2 &= y_{32} + y_{31,3} - (y_{12} + y_{13,1}) \\
&= y_{31,3} - y_{12} - y_{13,1} \qquad (35) \\
X_3 &= y_{13} + y_{12,1} - (y_{23} + y_{21,2}) \\
&= y_{13} + y_{12,1} - y_{21,2} \qquad (36)
\end{aligned}
$$

Substituting from equations (29)-(33)

$$
\begin{aligned}
X_1 &= \phi_1^* - \phi_{1,22}^* + \hat{n}_2 \cdot (\Delta_1^* - \Delta_{1,22}^*) + gw_2 + gw_{2,2} - \phi_1^* + \phi_{1,33}^* - \hat{n}_2 \cdot (\Delta_1^* - \Delta_{1,33}^*) - gw_3 - gw_{3,3} \\
&= \phi_{1,33}^* - \phi_{1,22}^* + \hat{n}_2 \cdot (\Delta_{1,33}^* - \Delta_{1,22}^*) + gw_2 + gw_{2,2} - gw_3 - gw_{3,3} \qquad (37) \\
&\approx gw_2 + gw_{2,2} - gw_3 - gw_{3,3} \quad \text{(only if } L_2 = L_3) \qquad (38)
\end{aligned}
$$

$$
\begin{aligned}
X_2 &= \phi_{1,3}^* - \phi_{1,333}^* + \hat{n}_2 \cdot (\Delta_{1,3}^* - \Delta_{1,333}^*) + gw_{3,3} + gw_{3,33} \\
&\quad -\phi_{1,3}^* + \phi_{1,21}^* - \hat{n}_2 \cdot (\Delta_{1,3}^* - \Delta_{1,21}^*) + gw_3 - gw_{2,1} - gw_1 \\
&\quad -\phi_{1,21}^* + \phi_{1,311}^* - \hat{n}_2 \cdot (\Delta_{1,21}^* - \Delta_{1,311}^*) + gw_{2,1} - gw_{3,11} - gw_{1,1} \\
&= \phi_{1,311}^* - \phi_{1,333}^* + \hat{n}_2 \cdot (\Delta_{1,311}^* - \Delta_{1,333}^*) + gw_{3,33} - gw_{3,11} + gw_3 + gw_{3,3} - gw_1 - gw_{1,1} \qquad (39) \\
&\approx gw_3 + gw_{3,3} - gw_1 - gw_{1,1} \quad \text{(only if } L_3 = L_1) \qquad (40)
\end{aligned}
$$

$$
\begin{aligned}
X_3 &= \phi_{1,2}^* - \phi_{1,31}^* + \hat{n}_2 \cdot (\Delta_{1,2}^* - \Delta_{1,31}^*) - gw_2 + gw_{3,1} + gw_1 \\
&\quad +\phi_{1,31}^* - \phi_{1,211}^* + \hat{n}_2 \cdot (\Delta_{1,31}^* - \Delta_{1,211}^*) - gw_{3,1} + gw_{2,11} + gw_{1,1} \\
&\quad -\phi_{1,2}^* + \phi_{1,222}^* - \hat{n}_2 \cdot (\Delta_{1,2}^* - \Delta_{1,222}^*) - gw_{2,2} - gw_{2,22} \\
&= \phi_{1,222}^* - \phi_{1,211}^* + \hat{n}_2 \cdot (\Delta_{1,222}^* - \Delta_{1,211}^*) + gw_{2,11} - gw_{2,22} + gw_1 + gw_{1,1} - gw_2 - gw_{2,2} \qquad (41) \\
&\approx gw_1 + gw_{1,1} - gw_2 - gw_{2,2} \quad \text{(only if } L_3 = L_1) \qquad (42)
\end{aligned}
$$